ON THE EXACT SOLUTIONS OF THE PROBLEM OF THREE BODIES

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The celebrated exact solutions of the problem of three bodies discovered by Lagrange (1772; Œuvres 6, 272-334) in an investigation "de pure curiosité" as he said, are today of much astronomical interest both from the theoretical point of view, by their applications to the periodical solutions of the three-body problem, and from the practical point of view by their usefulness, in the main, for the description of the motion of the fourteen known asteroids of the Trojan group discovered from 1906 on.

These exact solutions are homographic, i.e., such that the configuration of the three particles at any time t is similar to the initial configuration. Furthermore, Lagrange has shown, by means of "une analyse délicate et assez compliqué", which takes about twenty pages of his paper, that these solutions are planar, i.e., the three bodies move in a fixed plane.

Several astronomers and mathematicians (Laplace,Levi-Civita, Whittaker,Carathéodory,Kurth,...) have simplified Lagrange's proofs and generalized some of his results for more than three bodies and for some non-gravitational fields of forces.But it seems to me that no one has conveniently shown the elemental character of that problem.

On account of this, I have considered of some value to come back again to that problem, following a method similar to that of my paper (Publicaciones del Observatorio Astronómico de La Plata, Serie Astronómica, XXV nº 2, 1959), in which I have only considered in detail Jacobi's inverse cube law, assuming now a law of attraction inversely proportional to any power r^{α} $(\alpha \neq 3)$ of the distance r, which also applies to the kinetic theory of gases.

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Discusión:

Altavista: Noto una diferencia en el planteo que ha hecho Ud. con respecto a lo que se debe entender como definición de solución homográfica. De acuerdo al texto de Wintner, el hecho de que la configuración debe mantenerse semejante a si misma implica la multiplicación de la dilatación por una matriz de rotación.

Cesco: Este problema puede tratarse con matrices,o más elementalmente, como lo hago yo, aplicando directamente la definición que dá, por ejemplo, Kurth: Una solución es homográfica si la configuración de las tres partículas en cada instante es semejante a la configuración inicial.

Altavista: ¿Cómo plantearía el caso de una solución homotética?

Cesco: Las soluciones homotéticas corresponden,simultáneamente,al caso en que la dilatación es constante.

Altavista: ¿Y en el caso de las dilataciones puras?

Cesco: En tal caso los vectores P_1 y P_2 que he definido, se mantienen constantes en magnitud y dirección.